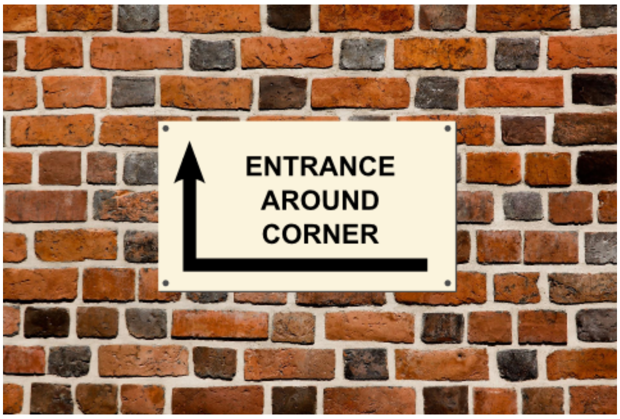
Mathematics B-day 2015 in Slovakia

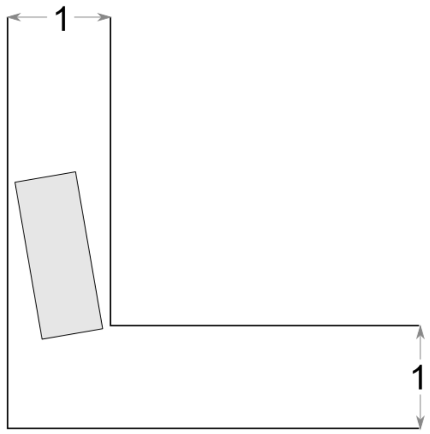
Tuesday, December 1, 9:00 h – 16:00 h

Around the corner…



SAMSUNG

Exploration 1 – piano



You have to move a heavy piano through a 1 meter wide corridor with a right-angled corner in it. The figure above shows the situation as seen from above. Can you manoeuvre the piano (1,54m long and 0,60m wide) around the corner? Examine this using, for example, a calculation, a drawing or an experiment.

Exploration 2 – sofa

As seen from above, a sofa is a rectangle that is 1,36m long and 0,78m wide. Can the sofa be manoeuvred around the corner?

Explanation

You have just seen two examples of rectangles, where you have probably already seen that sometimes you can, and sometimes you can't manoeuvre them around the corner, even when the rectangles have the same circumference.

Today you will further examine moving objects through the corridor. The question is whether certain objects can be moved through a 1 meter wide corridor with a right-angled corner in it. In other words, in today's assignments you will be looking at a moving problem **in two dimensions**; you will study which two-dimensional mathematical shapes (line segment, rectangle, circle, ...) can be moved through the corridor. As it is a problem in two dimensions, **the height of the objects is not considered**. From now on, the width of the corridor is 1 (without a unit). You can assume that objects with width equal to 1 fit precisely through the straight part of the corridor. So, a square with side 1 and a circle with diameter 1 can be moved through the corridor.

Mathematics B-day - general information

The structure of the day

This Mathematics B-day assignment consists of explorations, assignments and your own research. Try to spend approximately half of the day on your own research.

What to hand in?

You will hand in a report by the end of the day. In this report you describe the results you found in the *assignments* and *your own study*. You don't have to include the *explorations* in your report. Write your report in such a way that it is clear and convincing. Of course, you will use relevant explanatory images as illustrations. Make sure you report is understandable for people who did not take part in the Mathematics B-day. That means that you must take care to introduce the assignments clearly, and that, where needed, you must refer back to what you found in previous assignments.

In short: you write a clear and understandable report, supported by mathematical arguments. The quality of your report will definitely play a part in the assessment!

It may be useful for your report to already start writing down the assignments and answers you found during the morning. Keep in mind that the whole report has to be handed in by 4 o'clock in the afternoon!

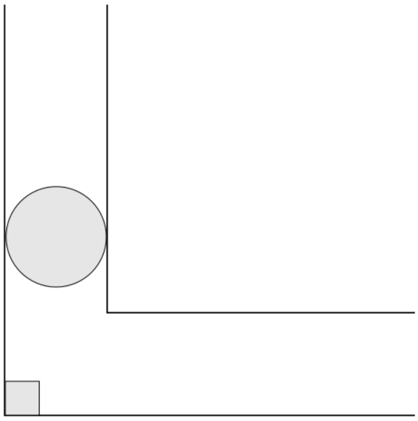
Information online

You will see that there is some information about this problem can be found online (for example, https://en.wikipedia.org/wiki/Moving_sofa_problem). However, in today's assignment you will look at other shapes than this 'sofa'.

Exploration 3 – half disk

a. What is the maximum radius for a semi-circular disk that you can move through the corridor?
b. Describe, as precise as you can, how to get the largest possible semi-circular disk through the corridor. For example: first put the semi-circular disk into position .., then move it over distance ..., then turn ... degrees around rotation point..., etc.

Exploration 4 – circle

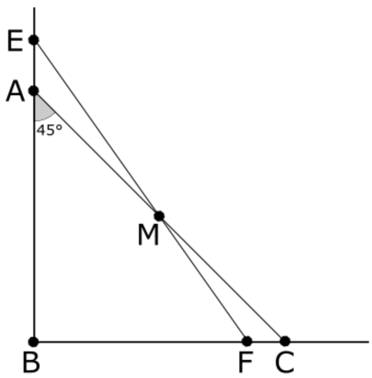


A circle with a diameter of 1 can be moved through the corridor. Now, suppose that there is a square obstructing the outer corner (see figure). What can be the maximum size of the square, such that the circle with a diameter of 1 can still be moved through the corridor?

Intermezzo

Before answering more assignments about moving objects through the corridor, here are two assignments that may be of use to you later today.

Assignment 1 – triangular geometry

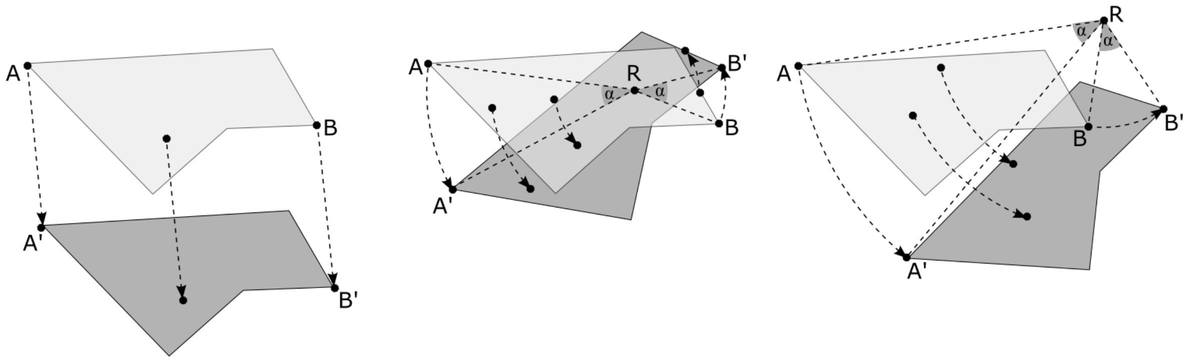


Given is an isosceles triangle *ABC*, where *M* is the midpoint of *AC*. Also, *F* is a point between *B* and *C* on the line *BC* and the line *FM* intersects the line *AB* in point *E*.
Show that line segment *EF* is longer than line segment *AC*.

Please note: even if you cannot prove it, you can still use the information that line segment EF is longer than line segment AC in later assignments.

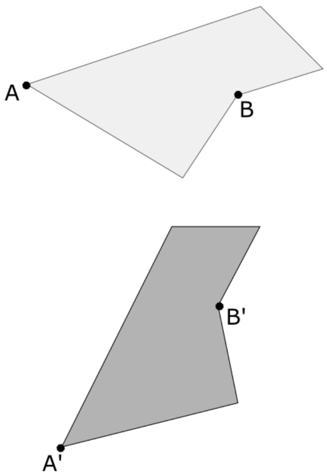
Assignment 2 – motions

You can move objects by sliding them (without rotating the object), or by rotating them around a centre (which we call R from now on). In the figure below an example of sliding and two examples of rotating are shown. For sliding, the following holds: $AA' = BB'$, plus AA' and BB' are parallel. For rotating, the following holds: $AR = A'R$, $BR = B'R$ and $\angle ARA' = \angle BRB'$.



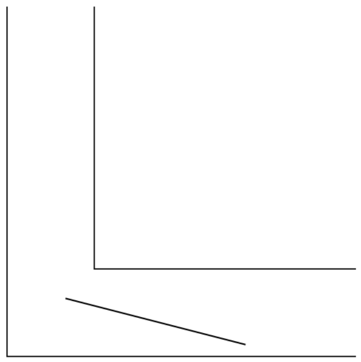
Please note: to perform a sliding motion in the corridor with the right-angled corner, all line segments (such as AA' and BB') must lie inside the corridor. For a rotation, all arcs (such as AA' en BB') must lie inside the corridor, but **point *R* does not necessarily have to lie inside the corridor**.

In the figure below you can also go from the light figure to the dark one by means of a rotation. Show how you can determine the centre R of the rotation.



Back to moving objects through a corridor...

Assignment 3 – Sticks



In this assignment you will look into which sticks can be moved around the corner, and which ones cannot. Consider a stick to be a line segment of a certain length.

a. Do some experiments (with transparent paper or a drawing) to try and discover which sticks can or cannot be moved around the corner.
b. With the aid of assignment 1 you can find out which sticks certainly can't be moved around the corner. Which ones are those? What is the length of the longest stick that can be moved around the corner?
c. Prove that you can in fact get the longest stick that you found in question b around the corner, and that it doesn't get stuck.

Assignment 4 –strategies for the longest stick

There are several ways to move the longest stick through the corridor.

a. In assignment 2 you looked at rotations. You can move the longest stick around the corner with a single rotation around a point *R*. Where is the rotation point *R* of this rotation, such that the longest stick can be rotated around the corner in this way? Also give a starting position from which you can make the rotation.
   Hint: Consider the moment that both ends of the stick touch the outside walls and the stick also touches the inner corner of the wall. If this is a snapshot of the rotation, you can find the circle that is described by the ends of the stick.
b. Make a sketch of the area covered by the stick in question a.
c. You can also perform a motion where the ends of the stick always touch the outside wall of the corridor. Describe the path that the middle of the stick follows when it is moved through the corridor this way as precise as possible.
d. Sketch the area covered by the stick in question c.

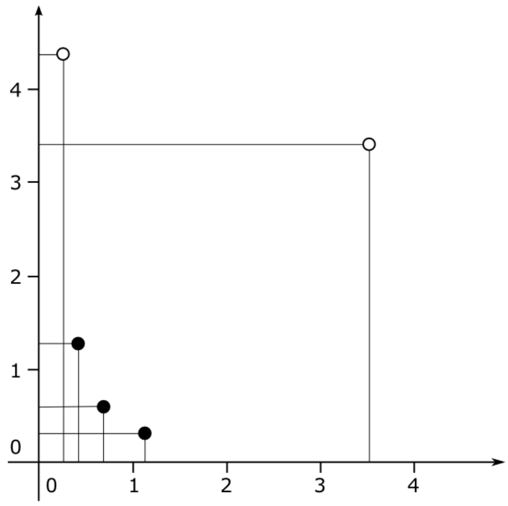
Assignment 5 – largest rectangle

In the first two explorations you already looked at rectangles. In this assignment you will look for the rectangle with the largest area that you can move through the corridor.

a. What is the area of the largest rectangle, with the shortest side of the rectangle being 0.5, that can be moved through the corridor?
b. Find the size of the rectangle with the largest area that you can move through the corridor by both sliding and rotating. The shortest side no longer has to be 0.5.
c. Show that the rectangle that you found in question b can actually be moved through the corridor, and does not get stuck in the corner.

Assignment 6 – all rectangles

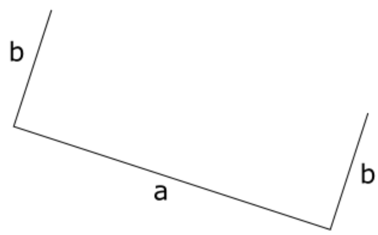
You have already looked at the largest rectangle that can be moved through the corridor. But what about all other rectangles? The figure below shows a coordinate system. In it, five rectangles have been drawn such that they have one side on the *x*-axis and one on the *y*-axis. The top right corners of the rectangles are black if the rectangle can be moved through the corridor and white if it cannot.



a. Where is the boundary between black and white points?
b. Which equations describe the boundary between black and white points?

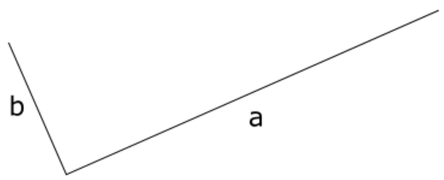
Assignment 7 – U-shape

You can attach two extra straight parts perpendicular to the ends of the longest stick, so that you get a U-shape. Find the maximum length of a U-shaped stick (length $a + 2b$ in the figure) that can be moved through the corridor.



Assignment 8 – L-shape

Consider a stick with one right angle (an L-shape). What is the longest stick (length $a+b$ in the figure) that can be moved through the corridor?



Your own study

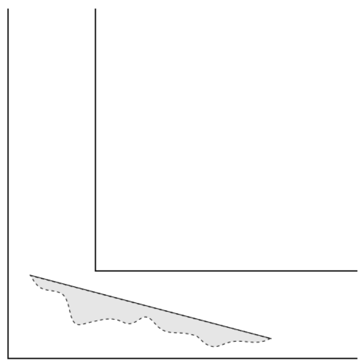
In the assignments so far you have already learned much about objects that can or cannot be moved through the corridor. Did you know that it is actually an **unsolved** problem what the largest object is that you can get through the corridor?

In this section you will further study objects that can be moved through the corridor. You can choose what to study. For example, you may choose a shape that has not yet been considered in the assignments so far. Questions you could ask yourself can concern:

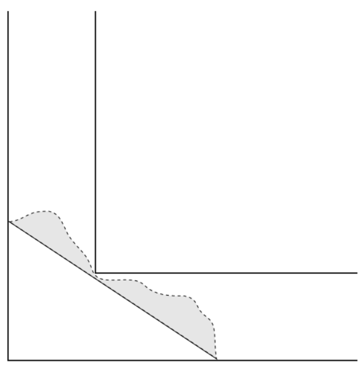
- maximum length or area of the object,
- different strategies to move the object through the corridor,
- the area that the object covers while being moved.

You can also continue with the research you have done so far. There are a few suggestions below. You can choose one, or you can be inspired by them:

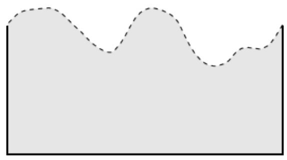
- Suppose that the longest stick is moved through the corridor as described in assignment 4a. You can add a lot of surface in the direction of the outer walls (see figure). Maybe even more if the stick isn't maximised! Study this..



- Suppose that the longest stick touches the outside walls, like in assignment 4c. You can add a lot of surface in the direction of the inner walls (see figure). Maybe even more if the stick isn't maximised! Study this.



- Suppose that you have a U-shape, as in assignment 7. You can add some extra surface on the inside of the U (see figure). Now, if the long (back) side of the U is moved along the outside walls of the corridor, how much surface can you add on the inside of the U? You could first consider the case where the long side is shorter than twice the width of the corridor. Can you also find - perhaps experimentally - the maximum shape if the long side is longer than twice the width of the corridor?



- Find a two-dimensional object that can be moved around the corner with one single rotation (similar to assignment 4a).